The role of the variational formulation in the hetero-dimensional and multiscale modeling of the cardiovascular human system

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As it is well known, the variational formulation is one of the most rational approaches to model an idealization of any real problem. In fact, a single integral expression contains all the information/requirements that the model must satisfy. Then, our aim in this work is to highlight some relevant roles played by variational formulations in several aspects of the modeling of the cardiovascular human system (CVS). For example, in the last 10 years several authors developed coupled dimensionally-heterogeneous models (3D-1D-0D) for the modeling of the CVS [1]. One of the issues in this approach is related to the coupling of these models [1, 2]. As we will show, an extended variational formulation is sufficient to tackle in a unified manner the kinematical discontinuities (due to the differences in the kinematical hypotheses adopted in each of these models), providing, in an automatic manner, the boundary conditions which are naturally satisfied at the boundary interfaces between such models [3]. Moreover, this variational formulation gives the appropriate framework for the study the sensitivity of the model to changes in the position of such interfaces. On the other hand, the mechanical behavior of biological tissues, and specifically arterial tissues, is really a multi-scale problem in which the configuration/behavior of elements such as elastin, collagen and smooth muscle among others play a fundamental role at micro-scale level, rendering the well-known complex behavior at the macro-scale level. Again a unified variational formulation allowing for a clear establishment of the theoretical basis that governs the above issue will be presented [4]. Although we deal with specific examples, these provide clear evidences that the variational approach results in a suitable framework for tackling the modeling and the computational simulation of this kind of physiological systems.

References

1. Formaggia L, Quarteroni A, Veneziani A. Cardiovascular Mathematics: Modeling and Simulation of the Circulatory System. Series: MS&A Springer-Verlag, 2009.
2. Formaggia L, Gerbeau JF, Nobile F, Quarteroni A. On the coupling of 3D and 1D Navier-Stokes equations for flow problems in compliant vessels. Comp. Meth. Appl. Mech. Engrg. 2001; 191:561-582.
3. Blanco PB, Feijóo RA, Urquiza SA. A uniffed variational approach for coupling 3D-1D models and its blood flow applications. Comp. Meth. Appl. Mech. Engrg. 2007; 196:4391-4410.
4. de Souza Neto EA, Feijóo RA; 'Variational Foundations of Multi-Scale Constitutive Models of Solid: Small and Large Strain Kinematical Formulation'; R&D LNCC's Report No. 16/2006.