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AN EXTENDED VARIATIONAL PRINCIPLE FOR COUPLING SHELL/PLATE MODELS AND 3D SOLID MODELS

*** Pablo J. Blanco[1] and Raúl A. Feijóo[1]**

[1] Laboratório Nacional de Computação Científica
Av. Getúlio Vargas 333, Quitandinha, 25651-075, Petrópolis, RJ, Brasil
{pjblanco,feij}@lncc.br

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ABSTRACT

The aim of this work is to present an extended variational formulation to handle the problem of coupling shell/plate models with 3D full solid models. Such an extended formulation has the purpose of accommodating the discontinuities that arise when admitting of the coexistence of incompatible kinematics to model a given structural component. The interest in using incompatible kinematics lies in the possibility of modeling the component partially as a simplified 2D model and partially as a 3D model. Another goal of this work is to present, together with the derivation of the corresponding extended variational formulation, the natural coupling conditions supplied by the governing principle. Although the main ideas are general, the analysis is restricted to the coupling of full 3D solid models with reduced 2D shell/plate models under the Kirchhoff–Love hypothesis for static analysis. In the next paragraphs the problem we are interested in and its context are described in a more detailed way.

On one hand, it has always been a common practice in structural analysis to perform hypotheses in order to make simplifications in the governing theory. Such hypotheses are based on the particularities of the problem, for instance by claiming that the displacement field in the deformation process owns a particular form. Thus it is possible to reduce the full 3D solid theory to 2D or even 1D theories so as to simplify the analysis. On the other hand, it has been of great interest the formulation of what was called junctions between structural components. In this respect, several works have dealt with the theoretical analysis and numerical approximation of junctions between plates [2], between shells [3,4] and also between 3D elastic structures and plates [1,7]. As well, a deeper mathematical analysis was carried out with the purpose of analyzing the problem of performing junction between components with different underlying dimensions [8-10].

Since a 2D, or even a 1D, structural component can always be understood as a 3D model that was reduced by incorporating suitable kinematical assumptions, the problem of performing the coupling between a 3D solid model and a reduced model, or equivalently, the problem of carrying out a junction of such models, can be interpreted from a different point of view from those ones used in the previously mentioned works. In such a standpoint it shall be accepted that in a given domain of analysis (the component under study) two different kinematics coexist, namely the one that allows the reduction of the model and the one corresponding to the 3D model. This entails that the two kinematics share a common internal boundary, that shall hereafter be regarded as the coupling interface between the models.

Over this artificial internal boundary a discontinuity in the displacement field arises as a result of the incompatibility between the underlying kinematics. Therefore, it is necessary to extend the governing principle in order to accommodate those discontinuities. The ideas behind this extension are simple and have their bases in the theory of Lagrange multipliers. Once the variational principle is extended we are able to work with non–matching kinematics for the same domain of analysis. This allows us to effectively perform the reduction of a portion of the domain to a simplified model and to obtain in a straightforward manner a formulation for the coupling between models of different dimensionality. One of the main interests in formulating the problem is the possibility of naturally extracting, from the associated Euler–Lagrange equations, the coupling conditions at the coupling boundary between the models without any further considerations. It shall be seen that the derivation of these natural coupling conditions permits us to characterize two distinguishable features concerning the behavior of the coupled problem concerning the quantities defined over the coupling interface. The concepts and ideas presented here have been previously developed, always in a variational framework, in [5,6]. In particular, in [6] a more general model that the one presented here was developed to account for the coupling of a 3D solid model and a 2D shell model under the Naghdi hypothesis, from which the Kirchhoff–Love case is a particular situation.

The structure of this work is the following: firstly we present the usual governing variational principle of solid mechanics and its extension using the ideas of kinematical incompatibility; secondly we introduce in the extended variational principle the Kirchhoff–Love hypothesis on a portion of the domain of analysis and formulate the problem of coupling a 3D solid with a shell; finally the shell is reduced to a plate, and the corresponding coupling problem between a 3D solid and a plate is automatically attained.

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